CUMULANT TO FACTORIAL MOMENT RATIO AND MULTIPLICITY DISTRIBUTIONS

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Abstract

The ratio of cumulant to factorial moments of multiplicity distributions has been calculated for e^+e^- and hh data in a wide range of energies. As a function of the rank the ratio exhibits a regular behaviour with a steep descent and two negative minima.

1. Introduction

Probing QCD predictions and phenomenological hypotheses upon the shape of multiplicity distributions is a major issue in Multiparticle Dynamics, and requires a careful comparison between experimental data and theory. Usually this task is accomplished through the study of the shape parameters of the distribution or through the examination of the behaviour of the factorial moments as a function of their rank. However, in a recent paper[1a] the possibility has been suggested of investigating multiplicity distributions through the ratio: $H_q = K_q/F_q$ of cumulant and factorial moments.

The q-th order factorial moments F_q and cumulant moments K_q can be easily obtained from any probability distribution P_n by means of the following relations

$$F_q = \Sigma_{n=1}^{\infty} n(n-1)...(n-q+1)P_n/ < n >^q , \qquad K_q = F_q - \Sigma_{m=1}^{q-1} C_{q-1}^m \, K_{q-m} \, F_m \ ,$$

with $F_0 = F_1 = K_1 = 1, K_0 = 0$ and where $C_{q-1}^m = q!/m!(q-m)!$. For the most common theoretical multiplicity distributions, the F_q's are characterized by absolute values rapidly increasing with the rank[1b] and display patterns that are not easy to distinguish from one another; the ratios H_q, on the contrary, under the same conditions lead to easily distinguishable patterns and remain bounded[1b,1c]. For instance: Negative Binomial Distribution (NBD) leads to positive defined H_q's, monotonically decreasing with the rank[1c]; similar features are expected by the QCD parton distribution in the Leading Logarithmic Approximation (LLA)[1c]; some additional os-

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Table 1: Investigated data.

e^+e^-	$\sqrt{s}$	hh	$\sqrt{s}$
TASSO[2a]	22 GeV	pp 30" B.C. at FNAL[3a]	23.8 GeV
HRS[2b]	29 GeV	pp SMF at CERN[3b]	30.4 GeV
TASSO[2a]	34.8 GeV	pp 30" B.C. at FNAL[3d]	38.8 GeV
TASSO[2a]	43.6 GeV	pp SMF at CERN[3b]	52.6 GeV
ALEPH[2c]	91 GeV	pp SMF at CERN[3b]	62.2 GeV
DELPHI[2d]	91 GeV	$\overline{p}p$ UA5[3e]	200 GeV
L3[2e]	91 GeV	$\overline{p}p$ UA5[3f]	546 GeV
OPAL[2f]	91 GeV	$\overline{p}p$ UA5[3e]	900 GeV

cillations are predicted by the Leading Double Logarithmic Approximation (DLA), the corresponding ratios still being, however, positive and globally decreasing[1c]. Predictions of a minimum of H_q at $q_{min_1} \simeq 4$ and indications of a further minimum at $q_{min_2} \simeq 2q_{min_1}$ are instead given by next-to-next to leading order gluodynamics when the non-linear terms in the equations for the generating functions of multiplicity distributions are properly treated[1d]. In this paper some results[1e,1f] on the behaviour of the ratio H_q for charged multiplicity from inelastic collision data are reported.

2. Experimental Results

Here we consider charged multiplicity data in full phase-space from e^+e^- annihilations[2] in the energy range $\sqrt{s} = 22$ to 91 GeV and from hh collisions[3] in the energy range $\sqrt{s} = 23.8$ to 900 GeV. The experiments are listed in Table 1. The results, up to the 16-th order, are given in fig.s 1 for e^+e^- experiments and in fig.s 2 for hh experiments; in each figure the experiments are sorted from top to bottom by increasing energy. The solid line interpolated to the points is meant only to guide the eye, for a better reading of the figures.

Most e^+e^- data display, as a function of the rank, an H_q behaviour characterized by a steep descent, taking place at the lower moment ranks, followed by a negative minimum, of order 10^{-4} and located between $q = 4$ and $q = 6$. Then H_q becomes positive, reaches a maximum and gives a second negative minimum between $q = 9$ and $q = 13$. In some of the figures also a second positive maximum is seen. This particular oscillatory trend is clear in the four LEP experiments, corresponding to fig.s 1e-1h, while for the TASSO and HRS data, fig.s 1a-1d show less pronounced and regular oscillations.

Qualitatively similar regularities are observed in the hh outcomes of fig.s 2, where an exponential descent, showing positive values, is followed by at least two negative minima. Here, however, the initial descent is less steep and the order of magnitude of the minima (between 10^{-3} and 10^{-2}) is larger than before.

3. Conclusions

Though we are at a preliminary approach to the whole problem, it is already clear that the proposed ratio H_q of cumulant and factorial moments is a sensitive measure of multiplicity distributions, since it helps in distinguishing various distributions which are hardly separated on the base of factorial moments only. The data presented here show that both in e^+e^- and hh full phase-space charged multiplicity distributions and in a wide energy range H_q displays, as a function of the rank, a rather regular behaviour with at least two negative minima. If a rough comparison between theory and data is performed one can observe that, experimental outcomes cannot be accounted for by NBD, LLA or DLA predictions; only next-to-next to leading gluodynamics predictions of ref. [1d] are in qualitative agreement with data. One could expect such qualitative features to be appropriate for e^+e^- annihilation since perturbative QCD is supposed to hold for hard processes and high energies, so it is a surprise to observe similar features also in soft hh collisions which are out of the scope of those theoretical approximations. Of course a comparison of the experimental behaviour of H_q with the results of ref [1d] is not proper on the quantitative level, because quarks, higher-order terms and confinement have not been considered there. Nonetheless the occurrence of the same qualitative features in so different interactions and energies deserves some theoretical attention and further experimental investigations.

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Figure 1: H_q vs. q for various e^+e^- experiments. On the left the first H_q orders in logarithmic scale, on the right the orders from q=3,4 to 16. The experimental outcomes are ordered according to their energy, that increases from top to bottom: (a) e+e- 22 GeV TASSO Coll.; (b) e+e- 29 GeV, HRS Coll.; (c) e+e- 34.8 GeV TASSO Coll.; (d) e+e- 43.6 GeV TASSO Coll.; (e) e+e- 91 GeV, ALEPH Coll.; (f) e+e- 91 GeV DELPHI Coll.; (g) e+e- 91 GeV OPAL Coll.; (h) e+e- 91 GeV, L3 Coll. (here the scale has been changed by a factor 2., with respect to the other experiments).

Figure 2: H_q vs. q for various hh experiments. On the left the first H_q orders in logarithmic scale, on the right the orders from q=3/4 to 16 (notice that the scale on the right is different from the one used in Fig.s 3); the points associated to exceedingly large error bars have not been plotted for clearness, only the corresponding portion of spline has been left. The experimental outcomes are ordered according to their energy, that increases from top to bottom: (a) pp 300 GeV/c (C.M.S. energy 23.8 GeV) FNAL 30 in. bubble chamber; (b) pp 30.4 GeV SMF det. at the CERN ISR; (c) pp 800 GeV/c (C.M.S. energy 38.8 GeV) E743 FNAL exp.; (d) pp 52.6 GeV SMF det. at the CERN ISR; (e) pp 62.2 GeV SMF det. at the CERN ISR; (f) $\overline{p}p$ 200 GeV UA5 Coll.; (g) $\overline{p}p$ 546 GeV UA5 Coll.; (h) $\overline{p}p$ 900 GeV UA5 Coll. .